Quantum Gravity: Puzzles and Perspectives

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See also: `arXiv:1301.5481`

Why Quantum Gravity?

- **Singularities in General Relativity (GR)**
  - **Black holes: gravitational collapse generically unavoidable**
  - **Singularity theorems: space and time 'end' at the singularity**
  - **Cosmological (big bang) singularity: what 'happened' at $t = 0$?**
  - **Structure of space-time at the smallest distances?**

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  - Perturbation theory: UV divergences in Feynman diagrams
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- **Difficulties probably have common origin:**
  - **Space-time as a continuum (differentiable manifold)**
  - **Elementary Particles as exactly *pointlike* excitations**
- **Expect *something* to happen at $\ell_{Planck} \sim 10^{-33} cm$!**

Different Attitudes

- **Hypothesis 1:**

  Quantum Gravity essentially *is* the (non-perturbative) quantization of Einstein Gravity (in metric/connection/loop or discrete formalism). Thus GR, suitably treated and eventually complemented by the Standard Model of Particle Physics or its possible extensions, correctly describes the physical degrees of freedom also at the very smallest distances.

- **Hypothesis 2:**

  GR is an effective (low energy) theory arising at large distances from a more fundamental Planck scale theory whose basic degrees of freedom are very different from either GR or QFT, and as yet unknown. GR, and with it, space-time itself as well as general covariance, are thus assumed to be 'emergent', much like macroscopic physics 'emerges' from the quantum world of atoms and molecules.

A Basic Fact

Perturbative quantum gravity is non-renormalizable

$$\Gamma_{div}^{(2)} = \frac{1}{\varepsilon} \frac{209}{2880} \frac{1}{(16\pi^2)^2} \int dV C_{\mu\nu\rho\sigma} C^{\rho\sigma\lambda\tau} C_{\lambda\tau}{}^{\mu\nu}$$

[Goroff& Sagnotti(1985); van de Ven(1992)]

Two possible conclusions:

- **UV divergences are artefacts of perturbative treatment ⇒ disappear upon a proper *non-perturbative* quantization of Einstein's theory; or**
- **Consistent quantisation of gravity requires a radical modification of Einstein's theory at short distances, in particular inclusion of supersymmetric matter.**

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No approach to quantum gravity can claim complete success that does not explain *in detail* the ultimate fate of this divergence and other divergences!

Gravity and Matter [→ Hermann Weyl (1918)]

Einstein's equations according to Einstein:

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{Marble}} = \underbrace{\kappa T_{\mu\nu}}_{\text{Timber?}}$$

Question: **can we understand the r.h.s. geometrically?**

- ***Kaluza-Klein theories?***
- ***Supersymmetry and Supergravity?***

Gravity vs. quantum mechanics: **do we need to change the rules of quantum mechanics?**

- ***Black hole evaporation and information loss?***
- ***Emergent space and time vs. quantum non-locality?***

Scales and Hierarchies

Gravitational force is much weaker than matter interactions $\Rightarrow$ the 'Hierarchy Problem'. The smallness of the Planck scale is *the* main obstacle towards experimental verification/falsification of any given ansatz!

This fact is also reflected in various mass scales

- **Known elementary particles cover a large mass range:** $m_\nu \sim 0.01\,\text{eV}\;,\; m_{\text{electron}} \sim 0.5\,\text{MeV}\;,\; m_{\text{top}} \sim 173\,\text{GeV}$
- **... but still tiny *vis-à-vis* Planck Scale** $M_{Pl} \sim 10^{19}\,\text{GeV}$ **.**
- **Cosmological constant:** $\Lambda_{pred}/\Lambda_{obs} \sim 10^{120}$**.**

A key challenge for any proposed theory of Quantum Gravity: offer *quantifiable* criteria to confirm or falsify the theory. These must in particular allow to discriminate the given proposal against alternative ones!

Approaches to Quantum Gravity

- Supergravity, Superstrings and M Theory
- AdS/CFT and Holography
- Path integrals: Euclidean, Lorentzian, matrix models,...
- Canonical Quantization (metric formalism)
- Loop Quantum Gravity
- Discrete Quantum Gravity: Regge calculus, (C)DT
- Discrete Quantum Gravity: spin foams, group field theory,...
- Non-commutative geometry and non-commutative space-time
- Asymptotic Safety and RG Fixed Points
- Causal Sets, emergent (Quantum) Gravity
- Cellular Automata ('computing quantum space-time')

Asymptotic Safety: is standard QFT enough?

[Weinberg(1979), Reuter (1995), Percacci(2006), Niedermaier(2007), Reuter&Saueressig(2012)]

Approach is closest in spirit to conventional QFT ideas (RG flow, RG group, etc.), but does not require anything special to happen to continuum space-time below ℓ_{Pl}! More specifically:

- **Is the UV limit of gravity determined by a non-Gaussian fixed point (NGFP) of the gravitational renormalisation group (RG) flow which controls the behaviour of theory at high energies and renders it safe from unphysical divergences?**

- **Aim: construct scale dependent effective action Γ_k**

  $$\lim_{k\to\infty} \Gamma_k = \text{bare action} \quad , \quad \lim_{k\to 0} \Gamma_k = \text{effective low energy action}$$

  $\Rightarrow$ approach is essentially *agnostic about microscopic theory*, all the information is in *universality classes* of RG flows.

- **M_{Planck} analogous to Λ_{QCD}: *lower end* of asymptotic scaling regime $\Rightarrow$ observable effects only if some prediction can be made about IR limit as theory flows down from NGFP.**

- **BUT:** [J.Donoghue, "A Critique of the Asymptotic Safety Program", arXiv:1911.02967]

Canonical Quantum Gravity

Non-perturbative and background independent approach: quantum metric fluctuations and quantum geometry.

- **Hamiltonian approach: manifest space-time covariance is lost through split ('foliation') of space-time as $\mathcal{M} = \Sigma \times \mathbb{R}$.**
- **$\to$ Space-time geometry is viewed as the *evolution of spatial geometry in time* according to Einstein's equations.**
- **Geometrodynamics: canonical *dynamical* degrees of freedom**

$$g_{mn}(t, \mathbf{x}) \quad \text{and} \quad \Pi^{mn}(t, \mathbf{x}) = \frac{\delta \mathcal{S}_{\text{Einstein}}}{\delta \dot{g}_{mn}(t, \mathbf{x})}$$

- **Dynamics defined by *constraints* (via shift and lapse): Hamiltonian constraint $\mathcal{H}(\mathbf{x})$ and diffeomorphism constraints $\mathcal{D}_m(\mathbf{x})$ $\Rightarrow$ Wheeler-DeWitt equation $\mathcal{H}(\mathbf{x})\Psi = 0$.**
- **Quantum Constraint Algebra from classical Poisson algebra:**

$$\{\mathcal{D}, \mathcal{D}\} \sim \mathcal{D} \quad \{\mathcal{D}, \mathcal{H}\} \sim \mathcal{H} \quad \{\mathcal{H}, \mathcal{H}\} \sim \mathcal{D} \quad \text{[modulo anomalies]}$$

New Variables, New Perspectives?

- **New canonical variables: replace g_{mn} by connection**

$$A_m{}^a = -\frac{1}{2}\epsilon^{abc}\omega_{m\,bc} + \gamma K_m{}^a$$

[$\omega_{m\,bc}$ = spatial spin connection, $K_m{}^a$ = extrinsic curvature]

- **New canonical brackets** [Ashtekar (1986)]

$$\begin{aligned} \{A_m{}^a(\mathbf{x}), E_b{}^n(\mathbf{y})\} &= \gamma\delta^a_b\delta^n_m\delta^{(3)}(\mathbf{x},\mathbf{y}) \;, \\ \{A_m{}^a(\mathbf{x}), A_n{}^b(\mathbf{y})\} &= \{E_a{}^m(\mathbf{x}), E_b{}^n(\mathbf{y})\} = 0 \end{aligned}$$

with conjugate variable $E_a{}^m$ = inverse densitized dreibein
$\Rightarrow$ for $\gamma = \pm i$ constraints become polynomial

$$E_a{}^n F_{mn}{}^a(A) \approx 0 \;, \quad \epsilon^{abc}E_a{}^m E_b{}^n F_{mn\,c}(A) \approx 0 \;, \quad D_m(A)E_a{}^m \approx 0$$

with $SU(2)$ field strength $F_{mna} \equiv \partial_m A_{na} - \partial_n A_{ma} + \varepsilon_{abc}A_m{}^b A_n{}^c$.

- **But reality constraint difficult to elevate to quantum theory $\rightarrow$ γ is nowadays taken real (‘Barbero-Immirzi parameter’)**

Loop Quantum Gravity (LQG)

- **Modern canonical variables: holonomy (along edge e)**

$$h_e[A] = \mathcal{P} \exp \int_e A$$

- **Conjugate variable = flux through area element S**

$$F_S^a[E] := \int_S dF^a = \int_S \epsilon_{mnp} E_a{}^m dx^n \wedge dx^p$$

- **act on wave functionals $\Psi_{\{\Gamma,C\}}[A] = f_C\Big(h_{e_1}[A], \dots, h_{e_n}[A]\Big)$ with *spin network* Γ (graph consisting of *edges* e and *vertices* v).**

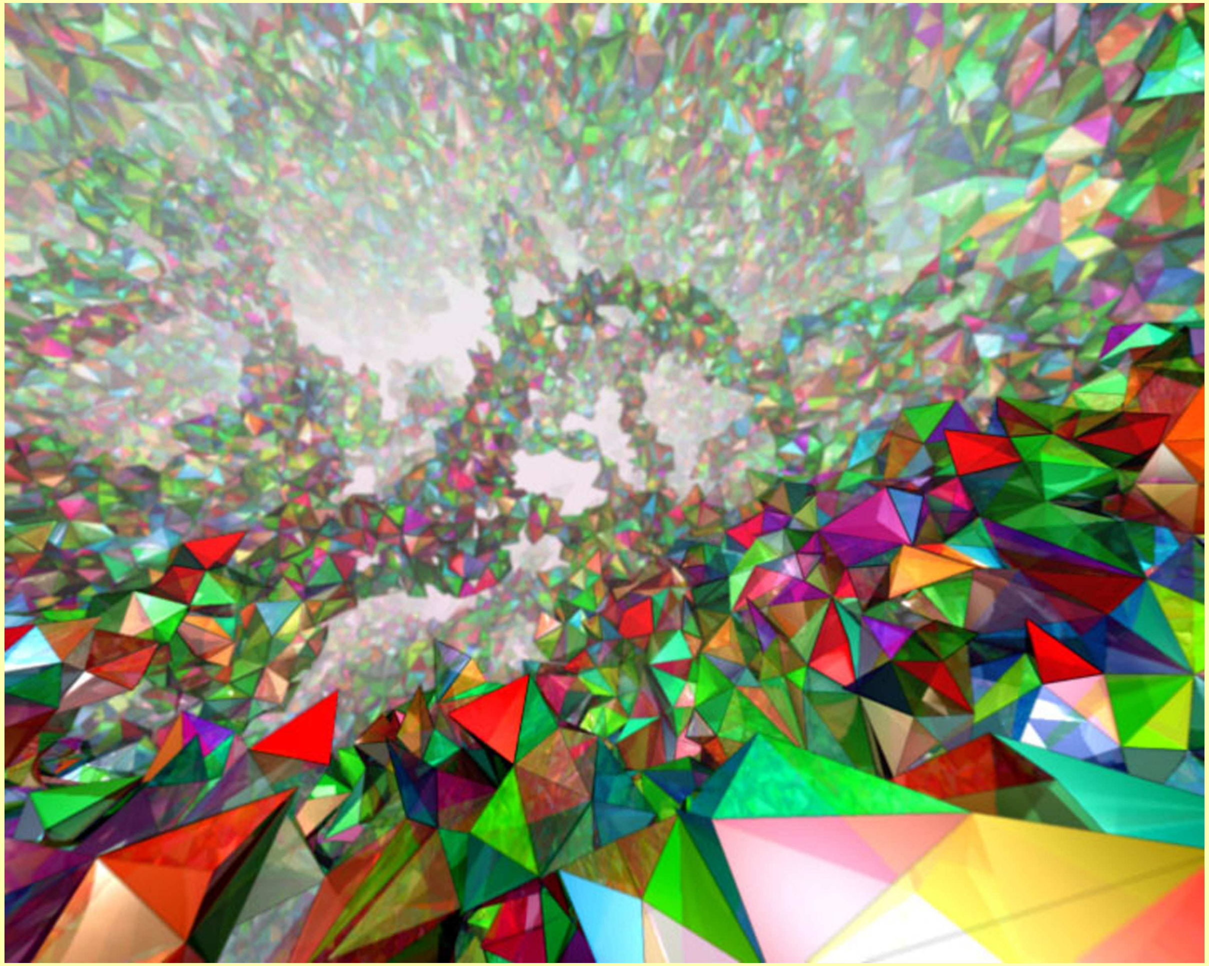
- **New feature: Kinematical Hilbert space $\mathcal{H}_{kin}$ can be defined, but is *non-separable* $\Rightarrow$ operators not weakly continuous.**

  Cf. ordinary quantum mechanics: replace $\langle x|x'\rangle = \delta(x - x')$ by $\langle x|x'\rangle = 1$ if $x = x'$ and $= 0$ if $x \neq x'$ $\rightarrow$ 'pulverize' real line!

- **$\Rightarrow$ No UV divergences (and thus no anomalies) ?**

- **$\Rightarrow$ No negative norm states ?** [cf. Narnhofer&Thirring (1992)]

LQG: a visualization of quantum space(-time)



Status of Hamiltonian constraint

- **Diffeomorphism constraint solved formally:** $\mathcal{X}_\Gamma = \sum_{\phi \in \mathrm{Diff}} \Psi_{\Gamma\circ\phi}$
- **$\Rightarrow$ Hamiltonian constraint not defined on $\mathcal{H}_{kin}$, but on distribution space $\mathcal{S}$ ('habitat') = dual of dense subspace $\subset \mathcal{H}_{kin}$.**
- **Main success: definition of regulated Hamiltonian (with $\epsilon > 0$) by means of kinematical operators (volume, etc.)** [Thiemann(2000)]

$$\hat{H}[N,\epsilon] = \sum_\alpha N(v_\alpha)\, \epsilon^{mnp} \mathrm{Tr}\Big(\big(h_{\partial P_{mn}(\epsilon)} - h^{-1}_{\partial P_{mn}(\epsilon)}\big)\, h_p^{-1} \left[h_p, V\right] \Big)$$
$$+ \frac{1}{2}(1+\gamma^2) \sum_\alpha N(v_\alpha)\, \epsilon^{mnp} \mathrm{Tr}\Big(h_m^{-1}\left[h_m, \bar{K}\right] h_n^{-1}\left[h_n, \bar{K}\right] h_p^{-1}\left[h_p, V\right] \Big)$$

- **Proper definition relies on diffeomeorphism invariance of states $\mathcal{X} \in \mathcal{S} \;\Rightarrow\;$ limit $\epsilon \to 0$ exists (at best) as a *weak limit*:**

$$\big\langle H^*[N]\mathcal{X} \big| \Psi \big\rangle = \lim_{\epsilon\to 0} \big\langle \mathcal{X} \big| \hat{H}[N,\epsilon]\Psi \big\rangle \;, \qquad \mathcal{X} \in \mathcal{S}$$

- ***Ultralocal* action of unregulated Hamiltonian adds 'spiderwebs' (of size $\epsilon \to 0$) to spin network Γ, but cumbersome to evaluate (on $\mathcal{S}$) even for the simplest examples.**

Summary and Critique

Non-perturbative approaches (LQG, spin foams, GFT) put main emphasis on general concepts underlying GR:

- (Spatial) Background Independence
- Diffeomorphism Invariance

However, these approaches so far do not incorporate essential insights and successes of standard QFT:

- Consistency restrictions from anomalies?
- Quantization ambiguities?
- Matter couplings: anything goes?

These issues will be hard to settle without a detailed understanding of how standard QFT and the semi-classical limit (Einstein equations, *etc.*) emerge.

The Superworld

Basic strategy: render gravity perturbatively consistent (i.e. finite) by modifying GR at short distances.

- **Supersymmetry: matter (fermions) *vs.* forces (bosons)**
- **(Partial) cancellation of UV infinities**
- **The *raison d'etre* for matter to exist?**
- **Maximally symmetric point field theories**
  - $D = 4, N = 8$ **Supergravity**
  - $D = 11$ **Supergravity**
- **Supersymmetric extended objects**
  - **No point-like interactions** $\Rightarrow$ **no UV singularities?**
  - **IIA/IIB und heterotic superstrings** ($D = 10$)
  - **Supermembranes and M(atrix)-Theory** ($D = 11$)

String Theory

Very much modelled on concepts from particle physics (hence no problem with semi-classical limit):

- **Not simply a theory of one-dimensional extended objects: D-branes, M-branes, ...**
- **Microscopic BH Entropy:** $S = \frac{1}{4}A$ **(+ corrections)**
- **Holography: the key to quantum gravity?**
- **New ideas for physics beyond the Standard Model:**
  - **Low energy supersymmetry and the MSSM**
  - **Large extra dimensions and brane worlds (but D = 4??)**
  - **Multiverses and the string landscape**

→ a new El Dorado for experimentalists?

String Theory: open questions

- **Struggling to reproduce SM *as is***
- **Struggling to incorporate** $\Lambda > 0$
- **Perturbative finiteness: obvious, but unprovable?**
- **Role of maximally extended** $N = 8$ **supergravity?**

Recent advances transcend perturbation theory, but

- **No convincing scenario for resolution of space-time singularities in GR (e.g. via AdS/CFT ?)**
- **Or: what 'happens' to space and time at** ℓ_{PL}**?**
- **The real question: what *is* string theory?**

Role of supersymmetry?

- **Are there alternatives to *low energy* ($N = 1$) supersymmetry to solve hierarchy problem?**
- **How is supersymmetry broken?**
  - **Can be arranged in supersymmetric field theories and ($N = 1$) supergravity models, though not very compellingly.**
  - **Problem is more acute and of more fundamental significance in superstring theory.**
- **Supersymmetry is *not* compatible with $\Lambda > 0$.**
- **Supersymmetry probably needed for consistent quantisation of gravity (cancellation of infinities,...), BUT:**
- **Spacetime supersymmetry *vs.* emergent spacetime: are there concepts that can 'supersede' supersymmetry? The hyperbolic Kac-Moody symmetry E_{10} 'knows everything' about maximal supersymmetry...**

A Key Issue: Non-Uniqueness

Existing approaches suffer from a very large number of ambiguities, so far preventing any kind of prediction with which the theory will stand or fall:

- **Superstrings:** 10^{500} **'consistent' vacua and the multiverse?**
- **LQG:** 10^{500} **'consistent' Hamiltonians/spin foam models?**
- **Discrete Gravity:** 10^{500} **'consistent' lattice models?**
- **Asymptotic Safety:** 10^{500} **'consistent' RG flows?**

Question: does Nature pick the 'right' answer at random from a huge variety of possibilities, or are there criteria to narrow down the number of choices?

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In order to discriminate between a growing number of diverging ideas on quantum gravity better to start looking for *inconsistencies*...

... or else *ansätze* may remain 'fantasy' [G.W. Gibbons]**!**

Forward to the Past: $N = 8$ Supergravity?

... *most symmetric* field theoretic extension of Einstein's theory of gravitation [Cremmer,Julia(1979); deWit,HN(1981)]

→ a promising candidate for the unification of all interactions with gravity? But:

- Existence of supersymmetric counter terms suggests non-renormalizable divergences from three loops onwards ⇒ no improvement over Einstein?
- Properties of theory (no chiral fermions, huge negative cosmological constant) in obvious contradiction to experiment and observation?

Last but not least: Superstring theory seemed to do much better in both regards...

$N = 8$ Supergravity: new perspectives

Very recent work has shown that $N = 8$ supergravity

- **is much more finite than expected (behaves like $N = 4$ super-Yang-Mills up to four loops)**

  [Bern,Carrasco,Dixon,Johansson, Roiban, PRL103(2009)081301]

- **However: recent computation at five loops shows divergence at $D = \frac{24}{5} = 2 + \frac{14}{L} < 4 + \frac{6}{L}$**

  [Bern,Carrasco,Chen,Edison,Johansson,Parra-Martinez,Roiban,PRD98(2018)086021]

But even if $N=8$ Supergravity is finite to all orders:

- **what about *non-perturbative* quantum gravity?**
- **is there any relation to *real physics*?**

Since no new spin-$\frac{1}{2}$ degrees of freedom have shown up at LHC, the following fact could become relevant:
(cf. recent work with Krzysztof Meissner)

A strange coincidence?

$SO(8) \to SU(3)\times U(1)$ **breaking and ‘family color locking’**

$(u\,,\,c\,,\,t)_L$:	$\mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}$,	$Q = \frac{2}{3} - q$
$(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L$:	$\bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}$,	$Q = -\frac{2}{3} + q$
$(d\,,\,s\,,\,b)_L$:	$\mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}$,	$Q = -\frac{1}{3} + q$
$(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L$:	$\bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}$,	$Q = \frac{1}{3} - q$
$(e^-,\mu^-,\tau^-)_L$:	$\mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}$,	$Q = -1 + q$
$(e^+,\mu^+,\tau^+)_L$:	$\mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}$,	$Q = 1 - q$
$(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L$:	$\mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}$,	$Q = -q$
$(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L$:	$\mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}$,	$Q = q$

$N = 8$ **Supergravity and Standard Model assignments agree if spurion charge is chosen as** $q = \frac{1}{6}$ [Gell-Mann (1983)]

Realized at SU(3) × U(1) stationary point. [Warner,HN: NPB259(1985)412]

Mismatch of $\pm\frac{1}{6}$ can be fixed by deforming U(1) [Meissner,HN:1412.1715]

Outlook

- **Incompleteness of the SM and GR are strongest arguments in favor of quantizing gravity.**
- **Main Question: how are short distance singularities resolved in GR *and* QFT, and how can this resolution be reconciled with classical Einstein equations in continuum space-time?**
  - **Dissolving pointlike interactions (strings, branes,...)**
  - **Cancellation of UV infinities (e.g. $N = 8$ supergravity)?**
  - **Fundamental discreteness (LQG, discrete gravity)?**
  - **Other mechanism (e.g. AS, non-commutative space-time)?**
- **Symmetry-based approach offers new perspectives: $N = 8$ supergravity and E_{10} are uniquely distinguished.**
- **... but there is still a long way to go!**